A Novel Intercarrier Interference Cancellation Approach in OFDM based on BSS

Yuan Liu and Wasfy Mikhael
Department of Electrical and Computer Engineering
University of Central Florida
Orlando, FL, 32816, USA

Abstract- Orthogonal Frequency Division Multiplexing (OFDM) is widely applied in wireless communication systems nowadays. In practice, there are frequency differences between the local oscillators in the transmitter and in the receiver. Due to these frequency offsets, the sub-carriers are not orthogonal. This leads to InterCarrier Interference (ICI) between the sub-carriers and degrades the system performance severely.

In this contribution, we propose a new ICI cancellation method based on Blind Source Separation (BSS). The relative gradient algorithm is employed to produce the separating matrix. Also, a technique to solve the permutation ambiguity is developed. The algorithm's interference cancellation is maintained for a wide range of interference conditions. Computer simulations are given, which confirm the effectiveness of the proposed compensation technique.

1. Introduction

In OFDM communication systems, a wideband source signal is partitioned into a number of the narrow sub-signals which are transmitted simultaneously employing the orthogonal sub-carriers. OFDM has various applications in wireless communications, such as digital audio, digital TV [1], and broadband satellite communication [2]. One major limitation of OFDM in many applications is its sensitivity to frequency differences between the local oscillators in the transmitter and in the receiver. These frequency offsets cause crosstalk between the sub-carriers, namely, ICI. ICI increases the Symbol Error Rate (SER) of the demodulated signal and degrades the performance of the receiver [3,4].

Zhao and Haggman have proposed ICI self-cancellation scheme to mitigate the ICI effects [5]. The resulting system is less bandwidth efficient than the normal OFDM scheme [5,6].

In this paper, an InterCarrier Interference Cancellation approach based on BSS (ICI/BSS) is proposed. In addition, a technique is given to solve the permutation ambiguity inherent in BSS type algorithms, which makes ICI/BSS practical in practice. Another advantage of ICI/BSS is that no prior information is needed. Also, no training sequences are required, which leads to efficient utilization of the bandwidth.

This paper is organized as follows. For a perfect Nyquist channel, the model of ICI resulting from the frequency offsets is described in detail in Section 2. ICI/BSS employing the relative gradient algorithm is given in Section 3. The technique to solve the permutation ambiguity is also given in Section 3. In Section 4, simulation results are given. Conclusions are drawn in Section 5.

2. Mathematical Formulation

The complete block diagram of the transceiver for OFDM system is given [1]. In this paper, the analysis considers only the impairments due to the frequency offsets. Thus, the forward error correct coding bock is not included. The frequency offsets alone do not cause InterSymbol Interference (ISI).

Usually, a cyclic prefix is used to eliminate the ISI in OFDM. The use of a cyclic prefix is not considered in this analysis too.

The structure of the simplified OFDM system, used to derive the ICI mathematical model, is shown in Fig. 1. In OFDM, interleaving is applied to correct bit errors, which occurs in burst rather being randomly scattered. A common used interleaving scheme is block interleaving, where input bits are written into a block column by column and read out row by row. The length of the block's column equals several times of the symbol length [7]. After interleaving, the serial source signal s(n) is independent at the time instant n. In general, s(n) is assumed to be complex-valued, zero-mean, stationary, and nongaussian distributed. The symbol time of s(n) is T_s. In Fig.1, the parallel source signals $s_i(m)$ (i=0,1,...,N-1) are given by

$$s_i(m) = s(iN + m) \tag{1}$$

where N is the number of the sub-carriers.

The symbol time of $s_i(m)$ is T_p, which is related to T_s as

$$T_p = NT_s \tag{2}$$

Since s(n) is white in the time domain, the signals $s_i(m)$ are considered to be mutually independent.

Then, N points Inverse Discrete Fourier Transform (IDFT) is applied to modulate $s_i(m)$ into the orthogonal sub-carriers. The modulated orthogonal signals $b_k(m)$ (k=0,1,...,N-1) are given by

$$b_k(m) = \frac{1}{N}\sum_{i=0}^{N-1} s_i(m)\exp\left(\tfrac{j2\pi ki}{N}\right) \tag{3}$$

where k is the IDFT index.

The signal b(n) is obtained from $b_k(m)$ by parallel to serial operation. The transmitted signal $s_T(t)$ is given by

$$s_T(t) = \exp(j2\pi f_c t)\sum_{n=-\infty}^{+\infty} b(n) g_T(t - nT_s) \tag{4}$$

where f_c is the local oscillator frequency, and $g_T(t)$ is the impulse response of the low-pass filter, in the transmitter.

To simplify the following analysis, the additive white gaussain noise n(t) is ignored in the derivation. The received signal $s_R(t)$ is given as

$$s_R(t) = s_T(t) \otimes h(t) \tag{5}$$

where h(t) is the impulse response of the channel and $\otimes$ denotes convolution.

In the receiver side, the local oscillator frequency is $f_c + \Delta f$, which is different from the transmitter side by Δf. The phase delay between the local oscillators in the transmitter and in the receiver is ignored. Thus, after the demodulation by $f_c + \Delta f$ and the low-pass filter $g_R(t)$, in the receiver, the signal $c(t)$ is given by

$$\begin{aligned} c(t) &= \exp(-j2\pi\Delta f t)\sum_{n=-\infty}^{+\infty} b(n) g_T(t - nT) \otimes h(t) \otimes g_R(t) \\ &= \exp(-j2\pi\Delta f t)\sum_{n=-\infty}^{+\infty} b(n) p(t - nT) \end{aligned} \tag{6}$$

where $g_R(t)$ is the impulse response of the low-pass filter in the receiver and p(t) is the combined impulse response of the transmitter filter, the channel filter, and the receiver filter.

Assuming that p(t) satisfies the Nyquist pulse-shaping criterion for samples taken at the optimum instants of intervals T_s [6]. After ADC and serial to parallel block in the receiver, the signals are

given by

$$c_k(m) = \exp\left(j2\pi\Delta f k T_s\right) b_k(m) = \exp\left(\tfrac{j2\pi ke}{N}\right) b_k(m) \tag{7}$$

where the frequency error is defined

$$e = N\Delta f T_s = \Delta f T_p \tag{8}$$

N points Discrete Fourier Transform (DFT) is applied to the signals, $c_k(m)$, to produce the demodulated signals

$$\begin{aligned} x_l(m) &= \sum_{k=0}^{N-1} c_k(m) \exp\left(\tfrac{-j2\pi kl}{N}\right) \\ &= \frac{1}{\mathrm{N}} \sum_{i=0}^{N-1} s_i(m) \sum_{k=0}^{N-1} \exp\left(\frac{-j2\pi k(i-l+e)}{N}\right) \\ &= \frac{1}{\mathrm{N}} \sum_{i=0}^{N-1} \alpha_{l,i} s_i(m) \end{aligned} \tag{9}$$

where the coefficient $\alpha_{l,i}$ is given by

$$\alpha_{l,i} = \frac{\sin\left[\pi\left(i-l+e\right)\right]}{N\sin\left[\frac{\pi(i-l+e)}{N}\right]} \exp\left[j\pi\left(\tfrac{N-1}{N}\right)\left(i-l+e\right)\right] \tag{10}$$

The relationship between the demodulated signals $x_l(m)$ and the parallel source signals $s_i(m)$ can be expressed in matrix format as follows

$$X(m) = AS(m) \tag{11}$$

where

$$X(m) = \left[x_0(m), \ldots, x_l(m), \ldots, x_{N-1}(m)\right]^T \tag{12}$$

$$S(m) = \left[s_0(m), \ldots, s_i(m), \ldots, s_{N-1}(m)\right]^T \tag{13}$$

and the mixing matrix is given by

$$A = \begin{pmatrix} \alpha_{0,0} & \cdots & \alpha_{0,N-1} \\ \vdots & \ddots & \vdots \\ \alpha_{N-1,0} & \cdots & \alpha_{N-1,N-1} \end{pmatrix} \tag{14}$$

If there are no frequency offsets, $\Delta f = 0$, $\alpha_{l,i} = 1 \ (l = i)$ and $\alpha_{l,i} = 0 \ (l \neq i)$. Thus, the matrix A is the identity matrix. Thus, no ICI occurs. If $\Delta f \neq 0$, $\alpha_{l,i}$ $(i \neq l)$ will not equals zero, which results in ICI. The value of $\alpha_{l,i}$ depends on the frequency error e and on $(i-l) \bmod N$, where mod is modulus after division, instead of directly on l and i. Hence, the mixing matrix A is a circulant matrix, and the inverse matrix of A is also a circulant matrix [8]. Also, all the diagonal elements of A are equivalent. Usually, in practice the frequency offsets are very small. Hence, $e \ll 1$ and

$$\left|\alpha_{i,i}\right| \gg \left|\alpha_{i,l}\right|, i \neq l \tag{15}$$

In other words, the absolute values of the diagonal elements in the matrix A are greater than the

off-diagonal ones. For the wide ranges of the frequency error, $-0.4 \le e \le 0.4$, and of the sub-carrier number, $4 \le N \le 52$, the absolute value of the diagonal element and the maximum absolute value of the off-diagonal elements are shown in Fig. 2. It is obvious that equ. (15) is generally valid in practice. This property is also held for the inverse matrix A, which is shown on Fig. 3.

3. Proposed ICI/BSS Approach

Usually, whitening is first applied before the BSS algorithm. X(m) is whitened by a whitening matrix Q to produce a white measurement signal vector $\tilde{X}(m)$,

$$\tilde{X}(m) = QX(m) = D^{-\frac{1}{2}}EX(m) \tag{16}$$

where D is the diagonal matrix of eigenvalues and E is the matrix of the eigenvectors of the covariance matrix R_{XX} of $X(m)$.

After whitening, the mixing model is given by

$$\tilde{X}(m) = QAS(m) = \tilde{A}S(m) \tag{17}$$

where the new mixing matrix $\tilde{A}$ is orthonormal.

To estimate an orthonormal mixing matrix greatly reduces the complexity of the BSS algorithm afterwards [8]. Then, the relative gradient method is used to determine the separating matrix $\tilde{W}$, which transforms the white measurement signals $\tilde{x}_i(m)$ into the estimates of the source signals $\tilde{s}_l(m)$. The update rule for the separating matrix $\tilde{W}$ is given by [10]

$$\tilde{W} = \tilde{W} + \mu\left[I - E\left(f\left(\tilde{S}(r)\right)\cdot\tilde{S}(r)^{H}\right)\right]\times\tilde{W} \tag{18}$$

where μ is the convergence factor, I is the identity matrix, $E(\ )$ denotes expectation, and the superscript H denotes conjugate transpose. The estimate source signal vector

$$\tilde{S}(m) = \tilde{W}\tilde{X}(m) = \left[\tilde{s}_1(m),\ldots,\tilde{s}_l(m),\ldots,\tilde{s}_{N-1}(m)\right]^T \tag{19}$$

and the nonlinear function

$$f\left(\tilde{S}(m)\right) = [f(\tilde{s}_1(m)),\ldots,f(\tilde{s}_l(m)),\ldots,f(\tilde{s}_{N-1}(m))]^T \tag{20}$$

The advantage of the relative gradient method is its equivarient property, namely, the performance of the relative gradient method does not depend on the mixing matrix [11].

After each iteration of the relative gradient method, the separating matrix $\tilde{W}$ is made to be orthonormal by the symmetric orthonormalization method [9].

$$\tilde{W} = \tilde{W}\left(\tilde{W}^H\tilde{W}\right)^{-\frac{1}{2}} \tag{21}$$

The separating matrix corresponding to the original matrix A is given by

$$W = \tilde{W}Q \tag{22}$$

The BSS type algorithm is associated with the permutation and gain ambiguities, which need to be resolved in the application. In other words, the separating matrix W is related to the mixing matrix A by

$$WA = \Lambda P \tag{23}$$

where P is the permutation matrix and Λ is the diagnose matrix given by

$$\Lambda = \begin{pmatrix} \beta_0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \beta_{N-1} \end{pmatrix} \tag{24}$$

where g_i is the unknown complex number.

The technique to solve the permutation ambiguity is given as follows. If W is without the permutation ambiguity, the separating matrix W is also in same format as the mixing matrix A, namely,

$$|w_{i,i}| \square |w_{i,l}|, i \neq l \tag{25}$$

The row permutation is applied to the separating matrix W to change its format to satisfy equ. (25). In this way, the permutation is solved successfully, and the compensated source signals are given by

$$\hat{S}(m) = WX(m) \tag{26}$$

where the parallel compensated signal vector

$$\hat{S}(m) = [\hat{s}_0(m), ..., \hat{s}_i(m), ..., \hat{s}_{N-1}(m)]^T \tag{27}$$

To resolve the gain ambiguity, additional information is needed to be embedded into the data streams. This information is usually in the form of predefined symbol sequences, which is used at the receiver detector to solve the gain ambiguity [9,12]. The whole ICI/BSS compensation scheme is summarized in Table 1.

4. Simulation Results

The performance of the proposed algorithm is evaluated through the computer simulations. In the simulated OFDM system, the number of sub-carriers N equals 8. For each of the parallel source signals $s_i(m)$, 30,000 samples of the baseband QPSK modulated signals are used, which are the statistically independent. In the relative gradient algorithm, the convergence factor μ equals 0.04. Also, the nonlinear function $f(y) = |y|^2 y$ is a good choice, since the typical baseband digital communication signals are complex with a negative kurtosis [13]. The effectiveness of ICI/BSS is demonstrated by comparing SER of the compensated signal $\hat{s}(n)$ with SER of the uncompensated signal $\hat{s}'(n)$, which is obtained from $x_i(m)$ by parallel to serial operation.

For 10dB signal-to-noise ratio (SNR), SER of $\hat{s}(n)$ and $\hat{s}'(n)$ vs the frequency error is given as Fig. 4. It is obvious the performance degradation due to the frequency offsets from the SER curve of $\hat{s}'(n)$. At the point $e = 0$, SER of $\hat{s}(n)$ and $\hat{s}'(n)$ are equivalent, which means ICI/BSS does not degrade the system performance if there are not the frequency offsets. From $e = 0$ to $e = 0.2$, SER of $\hat{s}(n)$ is almost constant, which means the effect of ICI is successfully removed after the compensation of ICI/BSS.

Under the fixed frequency offset $e = 0.1$, SER of $\hat{s}(n)$ and $\hat{s}'(n)$ vs SNR is given as Fig. 5. Thus, under the different noise levels, ICI/BSS is still effective and can greatly improve the system performance.

5. Conclusions

In this paper, the ICI model due to the frequency offsets for the perfect Nyquist channel in OFDM is derived. A compensation scheme for ICI in OFDM employing the BSS algorithm, namely, ICI/BSS is proposed. A practical technique is presented to solve the permutation ambiguity. This approach utilizes the bandwidth efficiently and does not require any prior information about the system. The simulation results confirmed the excellent properties of ICI/BSS in compensating the frequency offsets over a wide range. Also, the performance was maintained under different noise levels.

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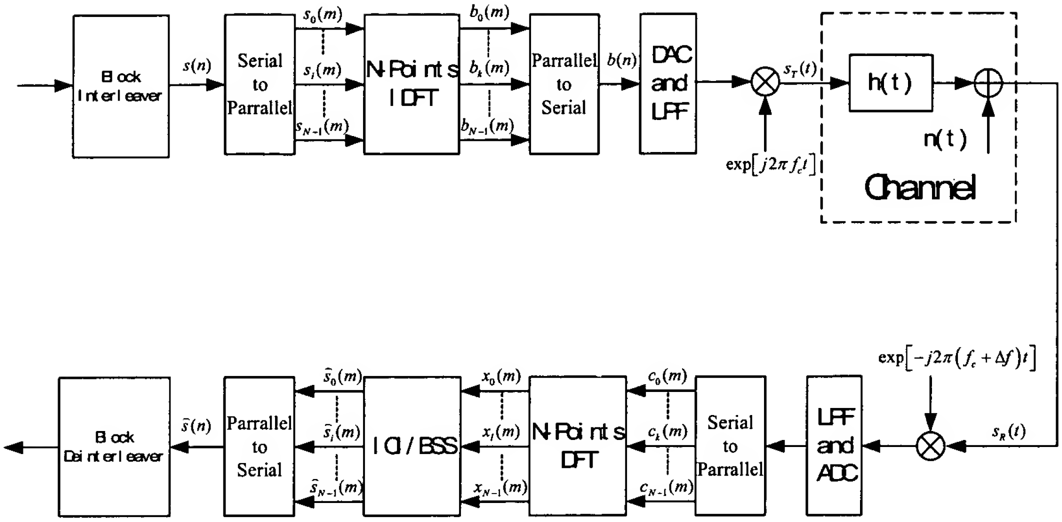
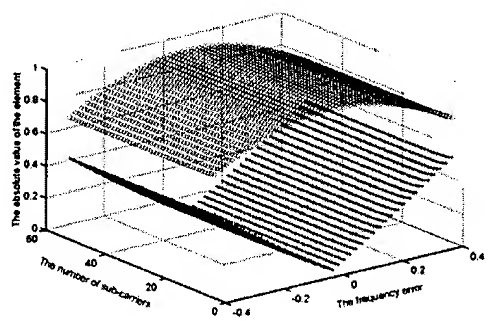


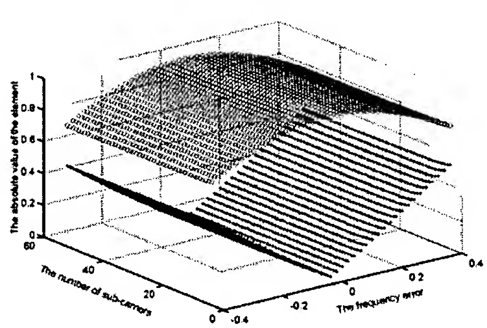
Fig.1: Structure of a simplified OFDM communication system



O The absolute value of the diagonal element

* The maximum absolute value of the off-diagonal elements

Fig.2: The absolute value of the diagonal and the maximum absolute value of the off-diagonal elements of the mixing matrix A, for the frequency error $-0.4 \leq e \leq 0.4$, and the sub-carrier number $4 \leq N \leq 52$



O The absolute value of the diagonal element

* The maximum absolute value of the off-diagonal elements

Fig.3: The absolute value of the diagonal and the maximum absolute value of the off-diagonal elements of the inverse matrix of the mixing matrix A, for the frequency error $-0.4 \leq e \leq 0.4$, and the sub-carrier number $4 \leq N \leq 52$

Step 1. Whiten of X(m) through a linear transform, equ. (16).

Step 2. Initialize the matrix $\tilde{W}$ as a random matrix.

Step 3. Update $\tilde{W}$ by the relative gradient algorithm to, equ.(18).

Step 4. Orthonormalize $\tilde{W}$, equ.(21).

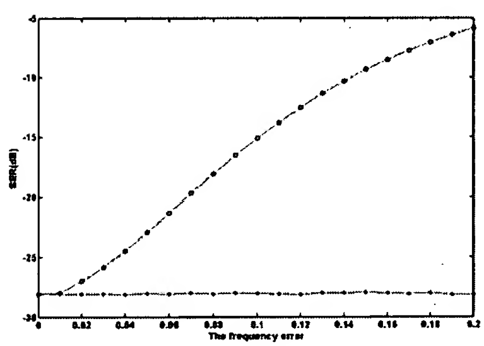
Step 5. Check the convergence of $\tilde{W}$, and if the convergence is not reached go back to Step 3, otherwise finish the iterations.

Step 6. Obtain the separating matrix W, equ.(22).

Step 7. Solve the permutation ambiguity of the separation matrix W.

Step 8. Obtain the compensated signal vector, equ.(26).

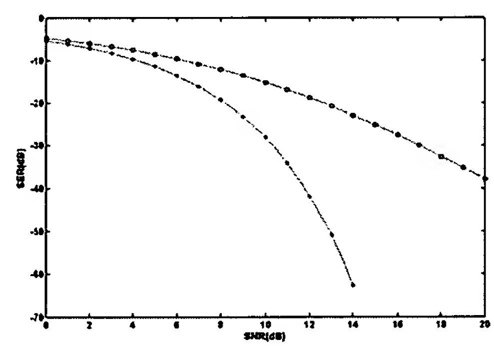
Table 1. Outline of the proposed ICI/BSS approach



O SER of $\hat{s}'(n)$

* SER of $\hat{s}(n)$

Fig.4: SERs vs. the frequency error



O SER of $\hat{s}'(n)$

* SER of $\hat{s}(n)$

Fig.5: SERs vs. SNR